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relativity

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THEORY OF RELATIVITY
An Introductory Sketch based on EINSTEIN'S Original Writings including a Biographical Note ☙
By HENRY L. BROSE, M.A.,
Published by Basil Blackwell, ☙
Broad St., Oxford. ☙ *mdccccxx.*

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BIOGRAPHICAL NOTE.

Albert Einstein was born in March, 1879, in the town Ulm, situated on the banks of the Danube in Würtemberg, Germany. He attended school at Munich, where he remained till his sixteenth year.

His university studies extended over the period 1896–1900 at Zürich, Switzerland. He became a citizen of Zürich in 1901. During the following seven years he filled the post of engineer in the Patent Office, Bern. He accepted a call to Zürich as Professor Extraordinarius in 1910, which he, however, soon resigned in favour of a permanent chair in Prague University. In 1911 he decided to accept a similar post in Zürich. Since 1914 he has continued his researches in Berlin as a member of the Berlin Academy of Sciences.

His most important achievements are:

1905. The Special Theory of Relativity.
The discovery that all forms of energy possess inertia.
The law underlying the Brownian movement.
The Quantum-Law of the emission and absorption of light.

1907. The fundamental notions of the general theory of relativity.

1912. The recognition of the non-Euclidean nature of space-determination and its connection with gravitation.

1915. Gravitational field equations.
Explanation of the motion of Mercury's perihelion.

PREFATORY NOTE.

The following pages represent the substance of a lecture which was given in the Clarendon Laboratory for the Science Colloquium. Friends who were present expressed the wish that the notes might be made available to a wider circle, and it is acting on this suggestion that I am venturing to have them published in pamphlet form. The object is to give a non-mathematical description of some of the leading ideas of the theory of relativity and to emphasise the successive steps in its development.

I wish to express my great indebtedness to Einstein's writings above all (the enunciations and definitions are taken from him), as well as to those of Weyl, Laue and Erwin Freundlich. A translation of a booklet by Freundlich entitled Die Grundlagen der Einsteinschen Gravitations theorie* is in course of publication by the Cambridge University Press, and will be found to fulfil the requirements of those who wish to avoid the cumbersome mathematics.

The philosophical aspects of relativity are clearly presented in a booklet by Schlick (oi Rostock), Raum und Zeit in der gegenwärtigen Physik,** an English version of which is also to be issued shortly by the Clarendon

* The Foundations of Einstein's Theory of Gravitation.
** Space and Time in contemporary Physics.

Press. The last chapter of his book discusses in detail how far the simultaneity of two events corresponds with the space-time coincidence of two elements of sensation. This question would exceed the bounds of an introductory sketch intended for the general reader.

My thanks are due to Dr. J. S. Haldane, F.R.S., for several helpful suggestions, and to Mr. H. W. Davies, M.B., B.S., for reading the proof sheets.

HENRY L. BROSE.

Christ Church, Oxford.

The second edition differs from the first in slight alterations on pages 21 and 30, which were suggested by criticisms of various friends and by a recent conversation with Einstein. The third (spectroscopic) test has not yet been definitely confirmed, but results hitherto obtained, if correctly interpreted, undoubtedly appear to support the theory. A valuable discussion on this point has been published by Dr. Freundlich in the periodical, *Die Naturwissenschaften* (Jul. Springer, Berlin), of 29th August, 1919. He indicates how difficult it is to analyse the complex Doppler shifts, which one observes in stellar spectra. Einstein considers this verification the most important of the three tests, as it is a direct outcome of the Principle of Equivalence, whereas the others depend on the form assumed for the equations of the gravitational field.

Feb. 2, 1920.

THE THEORY OF RELATIVITY.

INTRODUCTORY.

Physics, being a science of observation which seeks to arrange natural phenomena into a consistent scheme by using the methods and language of mathematics, has to inquire whether the assumptions implied in any branch of mathematics used for this purpose are legitimate in its sphere or whether they are merely the outcome of convention or have been built up from abstract notions containing foreign elements. The use of a unit length as an unalterable measure or of a time-division has been accepted in traditional mechanics without any inconsistency manifesting itself in general until the field of electrodynamics became accessible to investigators and rendered a re-examination of the foundations of our modes of measurement necessary. It is upon these that the whole science of mathematical physics rests. The road of advance of all science is in like manner conditioned by the inter-play of observations and notions, each assisting the other in giving us a clearer view of Nature regarded purely as a physical reality. The discovery of additional phenomena presages a still greater unification, revealing new relations and exposing new differences; the ultimate aim of physics would seem to consist in reaching perfect separation and distinctness of detail simultaneously with perfect co-ordination of the whole. "The all-embracing harmony of the world is the true source of beauty and is the real truth," as Poincaré has expressed it. The noblest task of co-ordinating *all* knowledge falls to the lot of philosophy.

A principle which has proved fruitful in one sphere of physics suggests that its range may be extended into others ; nowhere has this led to more successful results than in the increasing generalisation which has characterised the advance of the principle of relativity. This advance is marked by three stages, quite distinct, however, in the nucleus of their growth, yet the later one successively includes the *results* of the earlier.

Relativity first makes its appearance as a governing principle in Newtonian or Galilean mechanics : difficulties arising out of the study of the phenomena of radiations led to a new enunciation of the principle upon another basis by Einstein in 1905, which comprised the phenomena of both mechanics and radiation : this will be referred to as the " special " principle of relativity to distinguish it from the " general " principle of relativity enunciated by Einstein in 1915, which applied to all physical phenomena and every kind of motion. The latter theory also led to a new theory of gravitation.

1. The Mechanical Theory of Relativity.

In order to arrive at the precise significance of the principle of relativity in the form in which it held sway in classical mechanics, we must briefly analyse the terms which will be used to express it. Mechanics is usually defined as the science which describes how the " position of bodies in ' space ' alters with the ' time.' " We shall for the present only discuss the term " position," which also involves " distance," leaving time and space to be dealt with later when we have to consider the meaning of physical simultaneity.

Modern pure geometry starts out from certain conceptions such as "point," "straight line" and "plane," which were originally abstracted from natural objects and which are *implicitly* defined by a number of irreducible and independent axioms; from these a series of propositions is deduced by the application of logical rules which we feel compelled to regard as legitimate. The great similarity which exists between geometrically constructed figures and objects in Nature has led people erroneously to regard these propositions as true: but the truth of the propositions depends on the truth of the axioms from which the propositions were logically derived. Now empirical truth implies *exact* correspondence with reality. But *pure* geometry by the very nature of its genesis excludes the test of truth. There are no geometrical points or straight lines in Nature, nor geometrical surfaces; we only find coarse approximations which are helpful in representing these mathematically conceived elements.

If, however, certain principles of mechanics are conjoined with the axioms of geometry, we leave the realm of pure geometry and obtain a set of propositions which may be verified by comparison with experience, but only within limits, viz., in respect to numerical relations, for again no exact correspondence is possible, merely a superposition of geometrical points with places occupied by matter. Our idea of the form of space is derived from the behaviour of matter, which indeed conditions it. Space itself is amorphous, and we are at liberty to build up any geometry we choose for the purpose of making empirical content fit into it. Neither Euclidean,—nor any of the forms of meta-

geometry has any claim to precedence. We may select whichever is the more convenient for a consistent description of physical phenomena, and which requires a minimum of auxiliary hypotheses to express the laws of physical nature.

Applied geometry is thus to be treated as a branch of physics. We are accustomed to associate two points on a straight line with two marks on a (practically) rigid body: when once we have chosen an arbitrary, rigid body of reference, we can discuss motions or events mechanically by using the body as the seat of a set of axes of co-ordinates. The use of the rule and compasses gives us a physical interpretation of the distance between points and enables us to state this distance by measurement numerically, inasmuch as we may fix upon an arbitrary unit of length and count how often it has to be applied end to end to occupy the distance between the points. Every description in space of the scene of an event or of the positon of a body consists in designating a point or points on a rigid body imagined for the purpose, which coincides with the spot at which the event takes place or the object is situated. We ordinarily choose as our rigid body a portion of the earth or a set of axes attached to it.

Now Newton's (or Galilei's) law of motion states that a body which is sufficiently far removed from all other bodies continues in its state of rest or uniform motion in a straight line. This holds very approximately for the fixed stars. If, however, we refer the motion of the stars to a set of axes fixed to the earth, the stars describe circles of immense radius, that is, for such a system of reference the

law of inertia only holds approximately. Hence we are led to the definition of Galilean systems of co-ordinates. **A Galilean system is one, the state of motion of which is such that the law of inertia holds for it.** It follows naturally that Newtonian or Galilean mechanics is only valid for such Galilean or inertial systems of co-ordinates, *i.e.*, in formulating expressions for the motion of bodies we must choose some such system at an immense distance where the Newtonian law would hold. It will be noticed that this is an abstraction, and that such a system is merely postulated by the law of motion. It is the foundation of classical mechanics, and hence also of the first or "mechanical" principle of relativity.

If we suppose a crow flying in a straight line at uniform velocity with respect to the earth diagonally over a train likewise moving uniformly and rectilinearly with respect to the earth (since motion is change of position we must specify our rigid body of reference, viz., the earth), then an observer in the train would also see the crow flying in a straight line but with a different uniform velocity, judged from a system of co-ordinates attached to the train. We may consider both the train and the earth to be carriers of inertial systems as we are only dealing with small distances. We can then formulate the mechanical principle of relativity as follows :—

If a body be moving uniformly and rectilinearly with respect to a co-ordinate system K then it will likewise move uniformly and rectilinearly with respect to a second co-ordinate system K^1, provided that the latter be moving uniformly and rectilinearly with respect to the first system K.

In our illustration, the crow represents the body, K is the earth, and K^1 is carried by the train.

Or, we may say that if K be an inertial system then K^1, which moves uniformly and rectilinearly with respect to K, is also an inertial system. Hence, since the laws of Newtonian mechanics are based on inertial systems, it follows that all such systems are equivalent for the description of the laws of mechanics: no one system amongst them is unique, and we cannot define absolute motion or rest; any systems moving with mutual rectilinear uniform motion may be regarded as being at rest. Mathematically, this means that **the laws of mechanics remain unchanged in form for any transformation from one set of inertial axes to another.**

The development of electrodynamics and the phenomena of radiation generally showed, however, that the laws of radiation in one inertial system did not preserve their form when referred to another inertial system: K and K^1 were no longer equivalent for the description of phenomena such as that of light passing through a moving medium. This meant that either there was a unique inertial system enabling us to define absolute motion and rest in nature, or that we would have to build up a theory of relativity, *not* on the inertial law and inertial systems, but on some new foundation which would bring it about that the form of *all* physical laws would be preserved in passing from one system of reference to another.

This dilemma arose out of the conflicting results of two experiments, viz., Fizeau's and Michelson and Morley's.

Fizeau's experiment was designed to determine whether the velocity of light through moving liquid media was different from that through a stationary medium, *i.e.*, whether the motion of the liquid caused a drag on the aether, which it would do if the mechanical law of relativity held for light phenomena, for then the light ray would be in the same position as a swimmer travelling upstream or downstream respectively.*

No "aether-drag" was, however, detected, only a fraction of the velocity of the liquid seemed to be added to the velocity of light (c) under ordinary conditions and this fraction depends on the refractive index of the liquid and had previously been calculated by Fresnel: for a vacuum this fraction vanishes.

This result seemed to favour the hypothesis of a fixed aether, as was supported by Fresnel and Lorentz. But a fixed aether implies that we should be able to detect absolute motion, that is, motion with respect to the aether.

Arguing from this, let us consider an observer in the liquid moving with it. *If* there is a fixed aether, he should find a lesser value for the velocity of light (*i.e.*, $< c$) owing to his own velocity in the same direction or *vice versâ* in the opposite direction.

But we on the earth are in the position of the observer in the liquid since we revolve around the sun at the rate of approximately 30 kilometres per second (*i.e.*, $\frac{c}{10,000}$), and we are subject to a translatory motion of about the same magnitude: hence we should be able to detect a change in the velocity of

* It is well known that it takes a swimmer longer to travel a certain distance up and down stream than to swim across the stream and back an equal distance.

light due to our change of motion through the aether. These considerations gave rise to Michelson and Morley's experiment.

Michelson and Morley attempted to detect motion relative to the supposedly fixed aether by the interference of two rays of light, one travelling in the direction of motion of the earth's velocity, the other travelling across this direction of motion.

No change in the initial interference bands was, however, observed when the position of the instrument was changed, although such an effect was easily within the limits of accuracy of the experiment. Many modifications of the experiment likewise failed to demonstrate the presence of any "aether-wind."

To account for these negative results as contradicting deductions from Fizeau's experiment, Fitzgerald and, later, independently, Lorentz suggested the theory that bodies automatically contract when moving through the aether, and since our measuring scales contract in the same ratio, we are unable to detect this alteration in length; this effect would lead us always to get the same result for the velocity of light. This contraction-hypothesis agrees well with the electrical theory of matter and may be attributed to changes in the electro-magnetic forces, acting between particles, which determine the equilibrium of a so-called rigid body.

Thus Michelson and Morley's experiment seems to prove that the principle of relativity of mechanics also holds for radiation effects, that is, it is impossible to determine absolute motion through the aether or space: this implies that there is no unique system of coordinates. It disagrees with Fizeau's result

and seems to indicate the existence of a "moving aether," *i.e.*, an aether which is carried along by moving bodies, as was upheld by Stokes and Hertz. Lord Rayleigh pointed out that if the contraction-hypothesis of Lorentz and Fitzgerald were true, isotropic bodies ought to become anisotropic on account of the motion of the earth, and that consequently, phenomena of double refraction should make their appearance. Experiments which he himself conducted with carbon bisulphide and others carried out by Bruce with water and glass produced a negative result.

II. The "Special" Theory of Relativity.

Einstein, in the special theory of relativity, surmounts these difficulties by doing away with the aether (as a substance) and assumes that light-signals project themselves *as such* through space. Faraday had already long ago expressed the opinion that the field in which radiations take place must not be founded upon considerations of matter, but rather that matter should be regarded as singularities or places of a singular character in the field. We may retain the name "aether" for the field as long as we do not regard it as composed of matter of the kind we know. Einstein arrives at these conclusions by critically examining our notions of space and time or of distance and simultaneity

We know what simultaneity (time-coincidence of two events) means for our consciousness, but in making use of the idea of simultaneity in physics, we must be able to prove by actual experiment or observation

that two events are simultaneous according to some definition of simultaneity. **A conception only has meaning for the physicist if the possibility of verifying that it agrees with actual experience is given.** In other words, we must have a definition of simultaneity which gives us an immediate means of proving by experiment that, *e.g.*, two lightning-strokes at different places occur simultaneously for an observer situated somewhere between them or not. Whenever measurements are undertaken in physics two points are made to coincide, whether they be marks on a scale and on an object, or whether they be cross-wires in a telescope which have been made to coincide with a distant object and angular measurements made; *coincidence is the only exact mode of observation and lies at the bottom of all physical measurements.* The same importance attaches to simultaneity, which is coincidence in time. It is to be noted that no definition will be made for simultaneity occurring at (practically) one point: for this case psychological simultaneity must be accepted as the basis: the necessity for a physical definition only arises when two events happening at great distances apart are to be compared as regards the moment of their happening. We cannot do more than reduce the simultaneity of two events happening a great distance apart to simultaneity referred to a single observer at one point: this would satisfy the requirements of physics.

Einstein, accepting Michelson and Morley's result, introduced the convention in 1905 that light is propagated with a constant velocity ($= c$. *i.e.*, 300,000 km. per sec. approximately) in vacuo in all directions, and

he then makes use of light-signals to connect up two events in time.

He illustrates his line of argument roughly by assuming two points, A and B, very far apart on a railway embankment and an observer at M midway between A and B, provided with a contrivance such as two mirrors inclined at 90° and adjusted so that light from A and B would be reflected into his vertical line of sight (Fig. 1).

Two events such as lightning-strokes are then to be defined as simultaneous for the observer at M if rays of light from them reach the observer at the same moment (psychologically) : *i.e.*, if he sees the strokes in his mirror-contrivance simultaneously.

Next suppose that a very long train is moving with very great uniform velocity along the embankment, and that the lightning-strokes pass through the two corresponding points A^1 and B^1 of the train thus :

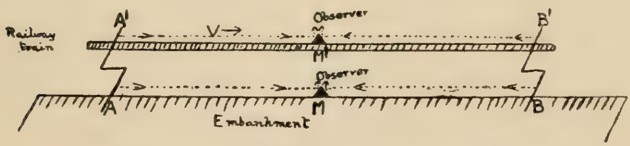

Fig. 1.

The question now arises: Are the two lightning-strokes at A and B, which are simultaneous with respect to the embankment also simultaneous with respect to the moving train ? It is quite clear that as M^1 is moving towards B^1 and away from A^1, the observer at M^1 (mid-point of A^1B^1) will receive the ray emitted from B^1 sooner than that emitted from A^1 and he would say that the lightning-stroke at B or B^1 occurred earlier than the one at A or A^1. Hence our condition of

simultaneity is not satisfied and we are forced to the conclusion that events which are simultaneous for one rigid body of reference (the embankment) are not simultaneous for another body of reference (the train) which is in motion with regard to the first rigid body of reference. This establishes the relativity of simultaneity.

This is, of course, only an elementary example of a very special case of the regulation of clocks by light-signals. It may be asked how the mid-point M is found: one might simply fix mirrors at A and B and by flashing light-signals from points between A and B ascertain by trial, the point (M) at which the return-flashes are observed simultaneously: this makes M the mid-point between the "*time*"-distance from A and B on the embankment.

The relativity of simultaneity states that every rigid body of reference (co-ordinate system) has its own time: a time-datum only has meaning when the body of reference is specified, or we may say that simultaneity is dependent on the state of motion of the body of reference.

Similar reasoning applies in the case of the distance between two points on a rigid body. The length of a rod is defined as the distance, measured by (say) a metre-rule, between the two points which are occupied *simultaneously* by the two ends. Since simultaneity, as we have just seen, is relative, the distance between two points, since they depend on a simultaneous reading of two events, is also relative, and length only has a meaning if the body of reference is likewise specified: any change of motion entails a corresponding change of length: we cannot detect the change

since our measures alter in the same ratio. **Length is thus a relative conception, and only reveals a relation between the observer and an object :** the "actual" length of a body in the sense we usually understand it does not exist : there is no meaning in the term. The length of a body measured parallel to its direction of motion will always yield a greater result when judged from a system attached to it than from any other system. These few remarks may suffice to indicate the relativity of distance.

In classical mechanics it had always been assumed that the time which elapsed between the happening of two events, and also the distance between two points of a rigid body were independent of the state of motion of the body of reference : these hypotheses must, as a result of the relativity of simultaneity and distance, be rejected. We may now ask whether a mathematical relation between the place and time of occurrence of various events is possible, such that every ray of light travels with the same constant velocity c whichever rigid body of reference be chosen, *e.g.*, such that the rays measured by an observer either in the train or on the embankment travel with the same apparent velocity.

In other words, if we assume the constancy of propagation of light in vacuo for two systems, K and K^1 moving uniformly and rectilinearly with respect to one another, what are the values of the co-ordinates x^1, y^1, z^1, t^1 of an event with respect to K^1, if the values x, y, z, t of the same event with respect to K are given ?

It is easy to arrive at this so-called Lorentz-Einstein transformation, *e.g.*, in the case

where K^1 is moving relative to K parallel to K's x axis with uniform velocity v.

$$x^1 = \frac{x - vt.}{\sqrt{1 - v^2/c^2}} \qquad y^1 = y \qquad z^1 = z$$

$$t^1 = \frac{t - v/c^2\, x}{\sqrt{1 - v^2/c^2}}$$

If we put $x = ct$, then we find that $\frac{x^1}{t^1}$ reduces to c.

i.e., $c = \frac{x}{t} = \frac{x^1}{t^1}$ is the same for both systems.

and the condition of the constancy of c, the velocity of light in vacuo, is preserved.

If $\sqrt{1 - v^2/c^2}$ is to be real, then v cannot be greater than c., *i.e.*, c is the limiting or maximum velocity in nature and has thus a universal significance.

If we imagine c to be infinitely great in comparison with v (and this will be the case for all ordinary velocities, such as those which occur in mechanics), the equations of transformation degenerate into :

$$x^1 = x - vt. \quad y^1 = y \quad z^1 = z \quad t^1 = t$$

This is the familiar Galilean transformation which holds for the " mechanical " principle of relativity. We see that the Lorentz-Einstein transformation covers both mechanical and radational phenomena.

The special theory of relativity may now be enunciated as follows :—**All systems of reference which are in uniform rectilinear motion with regard to one another can be used for the description of physical events with equal justification.** That is, if physical laws assume a particularly simple form when

referred to any particular system of reference, they will preserve this form when they are transformed to any other co-ordinate system which is in uniform rectilinear motion relatively to the first system. The mathematical significance of the Lorentz-Einstein equations of transformation is that the expression for the infinitesimal length of arc ds

$$(\text{viz., } ds^2 = dx^2 + dy^2 + dz^2 - c^2\, dt^2).$$

in the space-time * manifold x, y, z, t, preserves its form for all systems moving uniformly and rectilinearly with respect to one another.

Interpreted geometrically this means that the transformation is conformal in imaginary space of four dimensions. Moreover, the time-co-ordinate enters into physical laws in exactly the same way as the three space-co-ordinates, *i.e*, we may regard time spatially as a fourth dimension of space. This has been very beautifully worked out by Minkowski, whose premature loss is deeply to be regretted. It may be fitting to recall some remarks of Bergson in his *Time and Free Will.* He there states that "time is the medium in which conscious states form discrete series: this time is nothing but space, and pure duration is something different." Again, "what we call measuring time is nothing but counting simultaneities; owing to the fact that our consciousness has organised the oscillations of a pendulum as a whole in memory, they are first preserved and afterwards disposed in a series: in a word, we create for them a fourth dimension of space, which we call homogeneous time, and which enables the move-

* A continuous manifold may be defined as any continuum of elements such that a single element is defined by n continuously variable magnitudes.

ment of the pendulum, although taking place at one spot, to be continually set in juxtaposition to itself. Duration thus assumes the illusory form of a homogeneous medium and the connecting link between these two terms, space and duration, is simultaneity, which might be defined as the intersection of time and space." Minkowski calls the space-time-manifold "world" and each point (event) "world-point."

The results achieved by the special theory of relativity may be tabulated as follows:—

1. It gives a consistent explanation of Fizeau's and Michelson and Morley's experiment.
2. It leads mathematically at once to the value suggested by Fresnel and experimentally verified by Fizeau for the velocity of a beam of light through a moving refracting medium without making any hypothesis about the physical nature of the liquid.
3. It gives the contraction in the direction of motion for electrons moving with high speed, without requiring any artificial hypothesis such as that of Lorentz and Fitzgerald to explain it.
4. It satisfactorily explains aberration, *i.e.*, the influence of the relative motion of the earth to the fixed stars upon the direction of motion of the light which reaches us.
5. It accounts for the influence of the radial component of the light which reaches us, as shown by a slight displacement of the spectral lines of the light which reaches us from the

stars when compared with the position of the same lines as produced by an earth source.

6. It gives the expression for the increase of inertia, owing to the addition of (apparent) electromagnetic inertia of a charged body in motion.

The last result, however, introduces an anomaly inasmuch as the inertial mass of a quickly-moving body increases, but not the gravitational mass, *i.e.*, there is an increase of inertia without a corresponding increase of weight asserting itself. One of the most firmly established facts in all physics is hereby transgressed. This result of the theory suggested a new basis for a more general theory of relativity, viz., that proposed by Einstein in 1915. As the special theory of relativity only deals with uniform, rectilinear motions, its structure is not affected by any alteration of the ideas underlying gravitation.

III. The General Theory of Relativity.

We have seen that the first or "mechanical" theory of relativity was built up on the notion of inertial systems as deduced from the law of inertia; the "special" theory of relativity was built up on the universal significance and invariance of c, the velocity of light in vacuo; the third or general form of relativity is to be founded on the principle of the equality of inertial and gravitational mass and in contradistinction to the other two is to hold not only for systems moving uniformly and rectilinearly with respect to one another, but for all systems whatever their motion; *i.e.*, physical laws are to preserve their form for any arbitrary

transformaton of the variables from one system to another.

Mass enters into the formulae of the older physics in two forms: (1) Force = inertial mass multiplied by the acceleration. (2) Force = gravitational mass multiplied by the intensity of the field of gravitation.

$$\text{or}: \quad p = m.a \qquad p = m.^1 g$$

$$i.e., \quad a = \frac{m^1}{m} g$$

Observation tells us that for a given field of gravitation the acceleration is independent of the nature and state of a body; this means that the proportionality between the two characteristic masses (inertial and gravitational) must be the same for all bodies. By a suitable choice of units we can make the factor of proportionality unity, *i.e.*, $m = m^1$. This fact had been noticed in classical mechanics, but not interpreted.

Eötvös in 1891 devised an experiment to test the law of the equality of inertial and gravitational mass: he argued that if the centre of inertia of a heterogeneous body did not coincide with the centre of gravity of the same body, the centrifugal forces acting on the body due to the earth's rotation acting at the centre of inertia would not, when combined with the gravitational forces acting at the centre of gravitational mass, resolve into a single resultant, but that a torque or turning couple would exist which would manifest itself, if the body were suspended by a very delicate torsionless thread or filament. His experiment disclosed that the law of proportionality of inertial and gravitational mass is obeyed with extreme accuracy:

fluctuations in the ratio could only be less than a twenty-millionth.

Einstein hence assumes the exact validity of the law, and asserts that **inertia and gravitation are merely manifestations of the same quality of a body according to circumstances.** As an illustration of the purport of this equivalence he takes the case of an observer enclosed in a box in free space (*i.e.*, gravitation is absent) to the top of which a hook is fastened. Some agency or other pulls this hook (and together with it the box) with a constant force. To an observer outside, not being pulled, the box will appear to move with constant acceleration upwards, and finally acquire an enormous velocity. But how would the observer in the box interpret the state of affairs? He would have to use his legs to support himself and this would give him the sensation of weight. Objects which he is holding in his hands and releases will fall relatively to the floor with acceleration, for the acceleration of the box will no longer be communicated to them by the hand; moreover, all bodies will "fall" to the floor with the same acceleration. The observer in the box, whom we suppose to be familiar with gravitational fields, will conclude that he is situated in a uniform field of gravitation: the hook in the ceiling will lead him to suppose that the box is suspended at rest in the field and will account for the box not falling in the field. Now the interpretation of the observer in the box and the observer outside, who is not being pulled, are equally justifiable and valid, as long as the equality of inertial and gravitational mass is maintained.

We may now enunciate **Einstein's Principle of Equivalence: Any change which an ob-**

server perceives in the passage of an event to be due to a gravitational field would be perceived by him exactly in the same way, if the gravitational field were not present and provided that he—the observer—make his system of reference move with the acceleration which was characteristic of the gravitation at his point of observation.

It might be concluded from this that one can always choose a rigid body of reference such that, with respect to it, no gravitational field exists, *i.e.*, the gravitational field may be eliminated ; this, however, only holds for particular cases. It would be impossible, for example, to choose a rigid body of reference such that the gravitational field of the earth with respect to it vanishes entirely.

The principle of equivalence enables us theoretically to deduce the influence of a gravitational field on events, the laws of which are known for the special case in which the gravitational field is absent.

We are familiar with space-time-domains, which are approximately Galilean when referred to an appropriate rigid body of reference. If we refer such a domain to a rigid body of reference K^1 moving irregularly in any arbitrary fashion, we may assume that a gravitational field varying both with respect to time and to space is present for K^1 : the nature of this field depends on the choice of the motion of K^1. This enabled Einstein to discover the laws which a gravitational field itself satisfies. It is important to notice that Einstein does not seek to build up a model to explain gravitation but merely proposes a *theory of motions.* His equations describe the motion of any body in terms of co-ordinates of the space-time manifold,

making use of the interchangeability and equivalence implied in relativity. He does not discuss forces *as such* ; they are, after all, as Karl Pearson states "arbitrary conceptual measures of motion without any perceptual equivalent." They are simply intermediaries which have been inserted between matter and motion from analogy with our muscular sense.

A direct consequence of the application of the Principle of Equivalence in its general form is that the velocity of light varies for different gravitational fields and is only constant for uniform fields (this does not contradict the special theory of relativity, which was built up for uniform fields, and only makes it a special case of this much more general theory of relativity). But change of velocity implies refraction, *i.e.*, a ray of light must have a curved path in passing through a variable field of gravitation. This affords a very valuable test of the truth of the theory, since a star, the rays from which pass very near the sun before reaching us, would have to appear displaced (owing to the stronger gravitational field around the sun), in comparison with its relative position when the sun is in another part of the heavens : this effect can only be investigated during a total eclipse of the sun, when its light does not overpower the rays passing close to it from the star in question.* The calculated curvature is, of course, exceedingly small (1·7 seconds of arc), but nevertheless should be observable.

The motion of the perihelion of Mercury, discovered by Leverrier, which long proved an insuperable obstacle regarded in the light

* We shall return to this test at the conclusion of the chapter. (p. 29)

of Newtonian mechanics, is immediately accounted for by the general theory of relativity ; this is a very remarkable confirmation of the theory.

Before we finally enunciate the general theory of relativity, it is necessary to consider a special form of acceleration, viz., rotation. Let us suppose a space-time-domain (referred to a rigid body K) in which the first Newtonian Law holds, *i.e.*, a Galilean field : we shall suppose a second rigid body of reference K^1 to be rotating uniformly with respect to K, say a plane disc rotating in its plane with constant angular velocity. An observer situated on the disc near its periphery will experience a force radially outwards, which is interpreted by an external observer at rest relatively to K as centrifugal force, due to the inertia of the rotating observer. But according to the principle of equivalence the rotating observer is justified in assuming himself to be at rest, *i.e.*, the disc to be at rest. He regards the force acting on him as an effect of a particular sort of gravitational field (in which the field vanishes at the centre and increases as the distance from the centre outwards). This rotating observer, who considers himeslf at rest, now performs experiments with clocks and measuring-scales in order to be able to define time—and space—data with reference to K^1. It is easy to show that if, of two clocks which go at exactly the same rate when relatively at rest in the Galilean field K one be placed at the centre of the rotating disc and one at the circumference, the latter will continually lose time as compared with the former.

Secondly, if an observer at rest in K measure the radius and circumference of the

rotating disc, he will obtain the same value for the radius as when the disc is at rest, but since, when he measures the circumference of the disc, the scale lies along the direction of motion, it suffers contraction, and, consquently will divide more often into the circumference than if the scale and the disc were at rest. (The circumference does not change, of course, in rotation.) That is, he would get a value greater than π for the ratio $\frac{\text{circumference}}{\text{diameter}}$. This means that Euclidean geometry does not hold for the observer making his observations on the disc, and we are obliged to use co-ordinates which will enable his results to be expressed consistently. Gauss invented a method for the mathematical treatment of any continua whatsoever, in which measure-relations ("distance" of neighbouring points) are defined. Just as many numbers (Gaussian or curvilinear co-ordinates) are assigned to each point as the continuum has dimensions. The allocation of numbers is such that the uniqueness of each point is preserved and that numbers whose difference is infinitely small are assigned to infinitely near points. This Gaussian or curvilinear system of co-ordinates is a logical generalisation of the Cartesian system. It has the great advantage of also being applicable to non-Euclidean continua, but only in the cases in which infinitesimal portions of the continuum considered are of the Euclidean form. This calls to mind the remarks made at the commencement of this sketch about the validity of geometrical theorems. It seems as though the miniature view that we can take of straight lines and points in space led to a firm belief in the universal signifi-

cance of Euclidean geometry. When we deal with light phenomena which range to enormous distances, we find that we are not justified in confining ourselves to Euclidean geometry; the "straightest" line in the time-space-manifold is "curved." We must therefore choose that geometry which, expressed analytically, enables us to describe observed phenomena most simply: it is clear that for even large finite portions of space the non-Euclidean geometry chosen must practically coincide with Euclidean geometry.

We now see that the general theory of relativity cannot admit that *all rigid* bodies of reference K, K^1, etc., are equally justifiable for the description of the general laws underlying the phenomena of physical nature, since it is, in general, not possible to make use of rigid bodies of reference for space-time descriptions of events in the manner of the special theory of relativity. Using Gaussian coordinates, *i.e.*, labelling each point in space with four arbitrary numbers in the way specified above (three of these correspond to three space dimensions and one to time), the general principle of relativity may be enunciated thus:—

All Gaussian four-dimensional co-ordinate-systems are equally applicable for formulating the general laws of physics. This carries the principle of relativity, *i.e.*, of equivalence of systems to an extreme limit.

With regard to the relativity of rotations, it may be briefly mentioned that centrifugal forces can, according to the general theory of relativity, only be due to the presence of other bodies. This will be better understood by imagining an isolated body poised in space; there could be no meaning in saying that it

rotated, for there would be nothing to which such a rotation could be referred : classical mechanics, however, asserts that, in spite of the absence of other bodies, centrifugal forces would manifest themselves : this is denied by the general theory of relativity. No experimental test has hitherto been devised which could be carried out practically to give a decision in favour of either theory.

A favourable opportunity for detecting the slight curvature of light rays (which is predicted by the general theory) when passing in close vicinity to the sun occurred during the total eclipse of the 29th May, 1919. The results, which were made public at the meeting of the Royal Society on 6th November following, were reported as fully confirming the theory.

In addition to the slight motion of Mercury's perihelion, there is still a third test which is based upon a shift of the spectral line towards the infra-red, as a result of an application of Doppler's principle ; this has not yet led to a conclusive experimental result. Recent experiments by Dr. Freundlich, which are to be made known in the next issue of the *Physikalische Zeitschrift** will throw light on this question.

It is hoped that this short sketch of the main threads in the development of the subject will help to stimulate interest in a theory which revolutionises the views of classical physics and discloses the intimate connection between science and philosophy.

1. Note on Non-Euclidean Geometry.

In practical geometry we do not actually deal with straight lines, but only with distances, *i.e.*, with finite parts of straight lines, yet we feel irresistibly impelled to form some

* Vide *Physikalische Zeitschrift*, Dec, 1919,

conception of the parts of a straight line which vanish into inconceivably distant regions. We are accustomed to imagining that a straight line may be produced to an infinite distance in either direction, yet in our mathematical reasoning we find that in order to preserve consistency (in Euclid),* we may only allocate to this straight line *one* point at infinity: we say that two straight lines are parallel when they cut at a point at infinity *i.e.*, this point is at an infinite distance from an arbitrary starting-point on either straight line, and is reached by moving forwards or backwards on *either*.

Many attempts have been made, without success, to deduce Euclid's "axiom of parallels" which asserts that only one straight line can be drawn parallel to another straight line through a point outside the latter, from the other axioms. It finally came to be recognised that this axiom of parallels was an unnecessary assumption, and that one could quite well build up other geometries by making other equally justified assumptions.

If we consider a point, P, outside a straight line, L (Fig. 2) to send out rays in all directions, then, starting from the perpendicular position Pa_1 we find that the more obliquely the ray falls on L the further does the point of intersection a_n travel along L to the left (say). Our experience teaches us that the ray and L have one point in common. There is no justifiable reason, however, for asserting, as Euclid's axiom does, that for a final infinitely small increase of the angle a_1 Pa_n (*i.e.*, additional turn of Pa_n about P) a_n suddenly bounds off to infinity along L, *i.e.*, a_n, the point of intersection leaves finitude to

* According to the modern analytical interpretation of Euclid.

disappear into so-called "infinity," and that, for a further infinitesimal increase, a_n, reappears at infinity at the other end of L to the right of α_1.

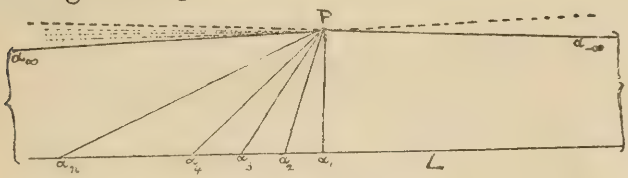

Fig. 2.

One might equally well assume, as Lobatschewsky did, that Pa_∞ and $Pa_{-\infty}$ form an angle which differs ever so slightly from two right angles, and that there are an infinite number of other straight lines included between these two positions (as indicated by the dotted lines in the figure), which do not cut L at all, Lobatschewsky (and also Bolyai) built up an entirely consistent geometry on this latter assumption.

Riemann later abolished the assumption of infinite length of a straight line, and assumed that in travelling along a straight line sufficiently far one finally arrives at the starting point again without having encountered any limit or barrier. This means that our space is regarded as being finite but *unbounded*.* Thus in Riemann's case there is *no* parallel line to L for a_n never leaves L; there is no a_∞. This geometry was called by Klein elliptical geometry (and includes spherical geometry as a special case). He calls Euclidean geometry parabolic (Fig. 3)

* *E.g.*, the surface of a sphere cuts a finite volume out of space but particles sliding on the surface nowhere encounter boundaries or barriers. This is a three-dimensional analogon to the *four* dimensional space-time-manifold of Minowski. It does not mean that the universe is enclosed by a spherical shell, as was supposed by the ancients. We cannot form a *picture* of the corresponding result in the four-dimensional continuum in which, according to the general theory of relativity, we live.

for the branches of a parabola continue to recede from one to another, and yet in order

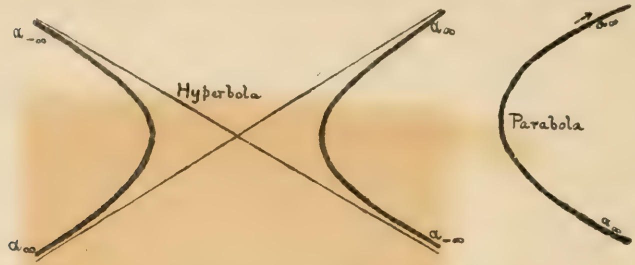

Fig. 3.

to obtain consistent results in its formulae we are obliged only to assign one point at infinity to it, just as to the Euclidean straight line. Lobatschewsky's geometry is similarly called hyperbolic (Fig. 4) since a hyperbola has two points at infinity, corresponding in analogy to the two points at infinity at which the two parallels through a point external to a straight line cut the latter.

The fact that one is obliged to renounce Euclidean geometry in the general theory of relativity leads to the conclusion that our space is to be regarded as finite but unbounded : it is curved, as Einstein expresses it, like the faintest of ripples on a surface of water : this point is discussed in detail by Schlick in his book mentioned in the preface.

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